Uplink Low-Power Scheduling for Delay-Bounded Industrial Wireless Networks Based on Imperfect Power-Domain NOMA

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***Abstract*—The power-domain non-orthogonal multiple access (NOMA) supports multiple packet reception, which can be leveraged for delay-bounded applications in industrial wireless networks (IWNs). However, it suffers from high power consumption on transmitters, which poses challenges for battery-powered wireless sensors. Given the delay bound for NOMA-based IWNs, the problem of minimizing aggregate power consumption of transmitters is therefore of great value. In a previous paper, we have addressed the problem under the model of perfect k-successive interference cancellation (k-SIC). In this paper, we study the same problem, however, under the model of imperfect k-SIC, which is more general in theory and more realistic in practice. For the existence of the optimal solution, we first present an explicit sufficient and necessary condition, which correlates three key parameters of network system together. We also propose a polynomial-time optimal algorithm with complexity $O(n^2)$. We further consider the same problem with discrete transmit powers, and present an approximation algorithm with complexity $O(n^2)$. Performance evaluation reveals that the delay bound requirement has tremendous impacts on both the aggregate power consumption and the maximum transmit power. Relative to the perfect SIC, the residual error caused by imperfect SIC results in extra power consumption of transmitters. However, the extra power consumption is gradually diminished with the further relaxation of the delay bound.**

***Index Terms*—Delay guarantee, low power, power control, successive interference cancellation (SIC), uplink scheduling.**

I. Introduction

In INDUSTRIAL wireless networks (IWNs), wireless sensors are usually deployed to sense the status of industrial processes and then, feedback the results to a sink. As a special case of wireless sensor networks, IWNs are usually cellular-style instead of *ad hoc*, because of the rigid requirements in industrial applications. Therefore, physical and MAC layer protocols with low latency, high reliability, and low power are research focuses in IWNs. Distinct from both the delay-tolerant wireless sensor networks and the delay-sensitive ultra-reliable low-latency communications (URLLC), some industrial applications, such as [2] and [3], are delay bounded. In other words, the UL (uplink) transmissions should be delay guaranteed for these applications in IWNs. Therefore, the guaranteed medium access delay is of vital importance to IWNs.[1]

Although the traditional time division multiple access (TDMA) has the advantage of providing delay guarantee, its medium access delay which equals the polling time would be high if there are massive wireless sensors. Relatively, since non-orthogonal multiple access (NOMA) can shorten the polling time drastically by supporting multiple parallel transmissions, it is suitable for delay-bounded applications in IWNs [4]. The power-domain NOMA, which is based on successive interference cancellation (SIC) receivers, is now under full consideration for industrial applications or heterogeneous cellular networks because of its support for massive connections [5]. However, tremendous electric energy on wireless sensors will be consumed for overcoming high interferences, which is an inherent shortcoming of the power-domain NOMA.[2] Obviously, the shortcoming poses great challenges for the battery-powered wireless sensors. Thus, the problem of minimizing the aggregate power consumption of wireless sensors[3] with delay guarantee for NOMA-based IWNs is of practical value.

To solve the problem, joint optimization of power controlling and UE scheduling is utilized. On one hand, the UE scheduling determines which UEs will transmit in parallel, i.e., how to group UEs. On the other hand, suitable transmit powers are set by power controlling so that signals of all parallel UEs can be decoded successfully by an SIC receiver.

Under the assumption of perfect SIC, i.e., there is no residual error for SIC, we design an efficient low-power scheduling scheme with complexity $O(n^2)$ in [1]. However, as we know, the perfect SIC is impossible in practice, and therefore, we need to

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[1]Generally speaking, the uplink transmission delay in IWNs consists of queuing delay and medium access delay. However, since the queuing delay is influenced by so many factors, and is hard to be precisely modeled and tracked, we only consider the medium access delay in this paper.

[2]In general, in power-domain NOMA, the power consumptions of transmitters increase exponentially with the enhancement of its parallel receiving capability. It can also be verified from Sections VII-A and VII-B.

[3]For convenience, in this paper, wireless sensor is abbreviated as UE (user equipment).

find valid algorithms for the same problem under the imperfect SIC.

Under imperfect SIC, since the residual errors left by canceling previous signals bring interferences to successive signals, the low-power scheduling problem is more complex than that under perfect SIC. Therefore, although the optimal solution to low-power scheduling is presented under perfect SIC model [1], we doubt the existence of the optimal solution under imperfect SIC. So, we conduct further studies in this paper, prove the existence of the optimal solution, and present some low-complexity algorithms for finding the optimal solution.

Our technical contributions are as follows.

1) We present an explicit sufficient and necessary condition for the existence of a feasible power solution for imperfect SIC. To the best of our knowledge, this paper presents the first feasibility result in the field of imperfect SIC.

2) We formulate the low-power scheduling problem as a mixed integer optimization problem by the joint UE scheduling and power allocation. By defining a key term named r-PTSI (power threshold sequence for imperfect r-SIC), we first reveal the structural characteristic of the optimal solution and then, propose a low-complexity optimal algorithm based on the structural characteristic.

3) We also propose an approximation algorithm with complexity $O(n^2)$ for the same problem with discrete transmit power levels, which must be dealt with for practicability,[4] and an approximation ratio is also presented.

Since both the model and the theoretic basis are distinct from those in [1], we declare that this paper is absolutely not a simple modification of [1], but a generalized extension to the imperfect SIC instead.

The remainder of this paper is organized as follows. Section II reviews the related works, and Section III presents the system models. Preliminaries are introduced in Section IV. Problem formulation and solution are introduced and analyzed in Section V. Based on the conclusions drawn in Section V, the same problem with discrete transmit powers is considered in Section VI. Performance evaluations are presented in Section VII, and conclusions are drawn in Section VIII.

II. Related Works

Nowadays, power-domain NOMA technologies receive extensive research efforts from wireless communications and networking fields [6]–[8]. The performance of power-domain NOMA is directly influenced by the capability of SIC receivers. Most early works focus on downlinks; we refer readers to [9] for an overview. As to the MAC protocols of the SIC-based network system, they can be categorized as random access-based and scheduling-based. For the random access-based algorithms, now there are three typical solutions, which are based on low-density parity-check codes [10], compressive sensing [11], and game theory [12], respectively. However, we only focus on the two related fields, i.e., scheduling-based low-power MAC algorithms and the imperfect SIC, in this paper.

1) Studies on the energy consumption in UL: Energy efficiency is an important aspect of power-domain NOMA [13]. Zhang *et al.* [14] reveal that to save UEs' power consumption, the transmit powers should be allocated based on the channel gains of UEs. In [6], fixed power allocation is introduced for two UEs such that the achieved UEs' rates are improved relative to the conventional orthogonal multiple access. We also minimize the aggregate power of UEs in [1] under the perfect k-SIC model, and present a tractable and optimal algorithm by means of a two-stage decomposition.

2) Studies of imperfect SIC: The imperfect SIC is now attaining more and more interests, where the linear residual error model, which is first discussed in 2003 [15], is being widely adopted [16]. Tweed *et al.* [17] optimize the aggregate power consumption of transmitters in the multichannel scenario, and an iterative convex optimization algorithm is utilized to solve the problem. Also, using the linear residual error model, Celik *et al.* [18] optimize the downlink capacity based on clustering and power-bandwidth tradeoff by formulating it as a mix-integer nonlinear programming problem. In [16], the fairness of transmit powers in UL MIMO-NOMA networks with imperfect SIC is formulated as a universal nonconvex optimization problem and solved by an approximation algorithm. We, however, present a closed-form solution to the aggregate power minimization problem on imperfect SIC, and propose a tractable optimal algorithm for the problem, instead of the time-consuming optimization-based algorithms.

III. System Model

We consider a single-hop, single-channel wireless network consisting of n single-antenna UEs[5] $u_1, u_2, \ldots, u_n$, and a single-antenna sink. The sink is equipped with a k-SIC receiver. A k-SIC receiver can decode at most k signals at one time, provided that SINR (signal-to-interference-plus-noise ratio) of every signal after interference cancellation is higher than the decoding threshold of the receiver. We assume that all of the n UEs have data to transmit.[6] Note that we focus on the uplinks, therefore, SIC receivers are not required for the UEs which are the transmitters. Fig. 1 plots a network example comprised of three UEs and a 2-SIC based sink.

The residual error of SIC is mainly caused by factors such as imperfect amplitude and phase estimation. Since they are closely related to the preamble power, the residual error is thought to be linear with the signal receive power, i.e., if the signal power

[4]The transmit power is not continuously adjustable for any transceiver nowadays. Take CC1000 which is manufactured by TI for example, there are 30 feasible power levels from – to dBm with a constant step size dBm. Therefore, if the transmit power calculated lies between two neighbor steps, in fact, it is infeasible for commercial-off-the-shelf radio transceiver.

[5]In this paper, UE, user, and transmitter are used interchangeably, and receiver is equivalent to sink.

[6]At the beginning of a frame, those UEs which have transmission tasks will report themselves to the sink via the control channel. Since we only need to find the UEs which try to be transmitters of the upcoming frame, methods such as [19], which is based on compressive sensing, can achieve the goal with low overhead.

$$\text{s.t.} \quad \frac{G_r p_r}{\sum_{i=1}^{r-1} G_i p_i + n_0} \geq \gamma \tag{1b}$$

$$\frac{G_l p_l}{\sum_{i=1}^{l-1} G_i p_i + \varepsilon \sum_{i=l+1}^{r} G_i p_i + n_0} \geq \gamma \quad \forall l \in [2, r-1] \tag{1c}$$

$$\frac{G_1 p_1}{\varepsilon \sum_{i=2}^{r} G_i p_i + n_0} \geq \gamma. \tag{1d}$$

Before we start to solve the MPAr PT problem, the following key definition is given.

Definition 2: Power threshold sequence for imperfect r-SIC (r-PTSI) is a sequence $\widehat{X}^{(r)} = (\widehat{X}_r^{(r)}, \widehat{X}_{r-1}^{(r)}, \ldots, \widehat{X}_1^{(r)})^T$ which satisfies the following group of equalities:

$$\begin{cases} \frac{\widehat{X}_r^{(r)}}{\sum_{i=1}^{r-1} \widehat{X}_i^{(r)} + n_0} = \gamma & \text{(2a)} \\ \frac{\widehat{X}_l^{(r)}}{\sum_{i=1}^{l-1} \widehat{X}_i^{(r)} + \varepsilon \sum_{i=l+1}^{r} \widehat{X}_i^{(r)} + n_0} = \gamma \quad \forall l \in [2, r-1] & \text{(2b)} \\ \frac{\widehat{X}_1^{(r)}}{\varepsilon \sum_{i=2}^{r} \widehat{X}_i^{(r)} + n_0} = \gamma & \text{(2c)} \end{cases}$$

where $\widehat{X}_i^{(r)} > 0$ for all $i \in [1, r]$ and $\gamma > 1$.

Obviously, $\widehat{X}_r^{(r)} \geq \widehat{X}_{r-1}^{(r)} \geq \cdots \geq \widehat{X}_1^{(r)}$, and all of them are

Thus, the RMLPSI-k SIC problem can be formulated as a mixed integer optimization problem

$$\min_{\{t_1,t_2,\ldots,t_n\},\{p_1,p_2,\ldots,p_n\}} \sum_{i=1}^{n} p_i \tag{3a}$$

corresponds to a feasible scheduling strategy. Now, Theorem 2 shows that the scheme mapped from the MWM is optimal.

Theorem 2: If and only if $\varepsilon\delta^{\lceil n/L \rceil} < 1$, Algorithm 1 outputs an optimal solution to RMLPSI-k SIC.

Proof. Please refer to Appendix E. ■

In Algorithm 1, we determine the optimal UE grouping strategy in line 10, and then, allocate optimal transmit powers for every slot in line 11. In other words, UE scheduling and power allocation in RMLPSI-k SIC could be decoupled,[16] and therefore, the computation complexity is greatly reduced. It is k-PTSI that separates power allocation and UE scheduling in the RMLPSI-k SIC problem without impairing the optimality. Thus, k-PTSI is key to the correctness of the optimal algorithm.

If the MWM in the tenth line of Algorithm 1 is found by Kuhn–Munkres algorithm [21], the time complexity of Algorithm 1 is $O(n^3)$ since that of Kuhn–Munkres algorithm is $O(n^3)$, and the complexity for setting up the graph is $O(n^2)$. Next, we present a faster algorithm for finding an MWM of the graph because it is a balanced complete bipartite graph.

Algorithm 2 is more complex than [1, Algorithm 2], where the perfect SIC is assumed. The intrinsic reason for the distinction of the two algorithms is that for any $i \in [1, r-1]$, $\widehat{X}_i^{(r)} = \widehat{X}_i^{(r-1)}$ holds for perfect SIC, while $\widehat{X}_i^{(r)} > \widehat{X}_i^{(r-1)}$ holds for imperfect SIC, which brings complexity in allocating slots for UEs.

Lines 1–4 of Algorithm 2 is for initialization, where $Phases$ saves all elements from both $\lfloor n/L \rfloor$-PTSI and $\lceil n/L \rceil$-PTSI. For any element of $Phases$, its $value$ is from $\lfloor n/L \rfloor$-PTSI if its $type$ is NF, otherwise, it is from $\lceil n/L \rceil$-PTSI if its $type$ is FU. In line 5, $Phases$ are reordered and saved to $Stphases$ in ascending order of the $value$ field. From lines 6–13, although it is implicit, we virtually construct n decoding positions T_{ij}, where $i \in [1, L]$ and $j \in \begin{cases} [1, \lfloor n/L \rfloor], & \text{if } i \in [1, L\lceil n/L \rceil - n] \\ [1, \lceil n/L \rceil], & \text{if } i \in [L\lceil n/L \rceil - n + 1, L] \end{cases}$, and the value of T_{ij} is $\begin{cases} \widehat{X}_j^{(\lfloor n/L \rfloor)}, & \text{if } i \in [1, L\lceil n/L \rceil - n] \\ \widehat{X}_j^{(\lceil n/L \rceil)}, & \text{if } i \in [L\lceil n/L \rceil - n + 1, L]. \end{cases}$ Then, the n UEs are mapped to the n decoding positions based on the principle that u_l should be mapped to the lth element in the ascending value of T_{ij}. In fact, the mapping is closely related with the ordering inequality theorem.

Theorem 3: Algorithm 2 outputs an MWM of $GH(n, k, L)$.

Proof. Please refer to Appendix F. The core of the proof is the ordering inequality theorem. ■

Theorem 3 reveals that for the optimal solution, the channel gain of any UE decoded in decoding phase i must be greater than that of any UE decoded in decoding phase $i+1$, where $i \in [1, k-1]$.

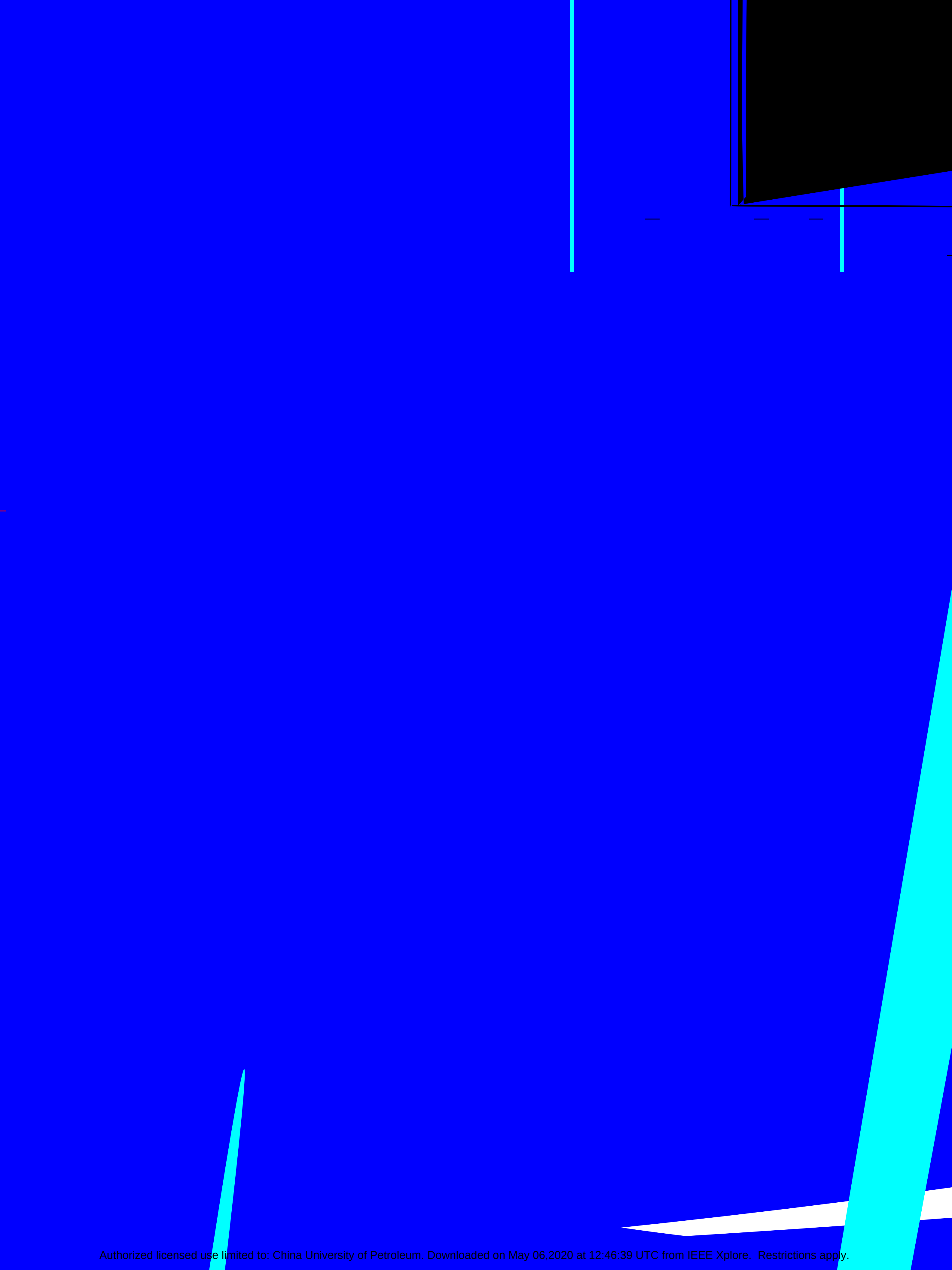
The complexity of Algorithm 2 is determined by the sorting algorithm used in line 5 of Algorithm 2. Generally, it is $O(n \log n)$

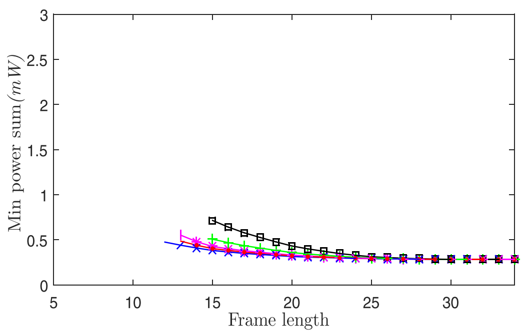
Algorithm 2: Faster Algorithm for MWM of $GH(n, k, L)${.

// Input: $GH(n, k, L), n, k, L$; Output: MWM of $GH(n, k, L)$;
struct phase {bool type; int phid; int value}:
Phases[$\lfloor n/L \rfloor + \lceil n/L \rceil$], Stphases[$\lfloor n/L \rfloor + \lceil n/L \rceil$];

1. $OUT = \varphi$;
2. sort $u_1, u_2, \ldots, u_n$ in the ascending order of their channel gains, WLOG, assume $G_1 \le G_2 \le \cdots \le G_n$;
3. for ($j = 1; j \le \lfloor n/L \rfloor; j{+}{+}$) {
   $Phases$[j].type=NF; $Phases$[j].phid=j;
   $Phases$[j].value=$\widehat{X}_j^{(\lfloor n/L \rfloor)}$;}
4. for ($j = 1; j \le \lceil n/L \rceil; j{+}{+}$) {
   $Phases$[j+$\lfloor n/L \rfloor$].type=FU;
   $Phases$[j+$\lfloor n/L \rfloor$].phid=j;
   $Phases$[j+$\lfloor n/L \rfloor$].value = $\widehat{X}_j^{(\lceil n/L \rceil)}$;}
5. sort $Phases$ based on its $value$ field in the ascending order, and save the ordered results into array $Stphases$;
6. idx=1;
7. for (ja n 1 3 1 0 c 1 9 9 9 2 8 3 ( ) 1 1 1

[16] First, Lemma 4 narrows the searching space. Second, Theorem 1 reveals that for any UE scheduling strategy, its optimal power allocation strategy is analytically known. Therefore, for any given UE scheduling strategy, we can compute its minimal aggregate power consumption. By comparing the minimal aggregate power consumption of these UE scheduling strategies which are in the above narrowed strategy space, we can find the optimal UE scheduling strategy. Obviously, with respect to that of the original blind searching method, its computation complexity will be reduced greatly.




Min power sum(mW)
Frame length
0
0.5
1
1.5
2
2.5
3
5
10
15
20
25
30

CPLEX are listed, where the value in the bracket is for CPLEX. According to the result, the execution time of Algorithm 1 is around 20 ms, and that of CPLEX is around 230 ms when the optimal UE scheduling strategy is given. So, we can conclude that the time for the traditional searching algorithm to solve RMLPSI-k SIC is much longer than that of Algorithm 1.

VIII. Conclusion

This paper focuses on the tradeoff between power consumptions and real-time performance requirements of uplink transmissions for imperfect SIC-based wireless networks. We solve this problem by developing optimal power scheduling algorithms. Our conclusions are as follows: 1) under a given sufficient and necessary condition, the problem is solvable in $O(n^2)$ time in the case of continuous transmit powers, and an optimal power scheduling strategy is obtained in this paper; 2) the requirement of real-time performance has a major impact on power consumption than other factors, such as the number of simultaneous transmitters supported by imperfect SIC receiver; and 3) under the same given condition, the problem in the case of discrete transmit powers can be solved by an approximation algorithm with time complexity of $O(n^2)$.

Although the power-domain NOMA is proposed for improving spectral efficiency, it is also suitable for delay-bounded applications in IWNs. By fine-grained power scheduling, low-power consumption performance under given delay guarantee can still be provided, which makes power-domain NOMA an ideal choice for IWNs.

Appendix

A. Proof of Lemma 2

Proof. Since $\left(\widehat{X}_r^{(r)}, \widehat{X}_{r-1}^{(r)}, \ldots, \widehat{X}_1^{(r)}\right)$ is r-PTSI. $\widehat{X}_j^{(r)} = \gamma(\sum_{i=1}^{j-1} \widehat{X}_i^{(r)} + \varepsilon\sum_{i=j+1}^{r} \widehat{X}_i^{(r)} + n_0)$. Therefore, $\widehat{X}_{j+1}^{(r)} = \frac{\gamma+1}{\gamma\varepsilon+1}\widehat{X}_j^{(r)}$, i.e., r-PTSI is a geometric sequence.

In (2c), by substituting $\widehat{X}_i^{(r)}$ with $\left(\frac{\gamma+1}{\gamma\varepsilon+1}\right)^{i-1}\widehat{X}_1^{(r)}$, $\widehat{X}_1^{(r)} = \frac{\gamma n_0}{\delta_{r-1}-\gamma\sum_{i=0}^{r-2}\delta^i}$, where $\delta = \frac{\gamma+1}{\gamma\varepsilon+1}$ and $r \geq 2$. So, $\widehat{X}_i^{(r)} > 0$ holds for $\forall i \in [1, r]$, if and only if $\frac{\gamma n_0}{\delta^{r-1}-\gamma\sum_{i=0}^{r-2}\delta^i} > 0$, or equivalently $\varepsilon\delta^r < 1$. In all, r-PTSI exists if and only if $\varepsilon\delta^r < 1$. ■

B. Proof of Lemma 3

Proof. 1) With the Gaussian elimination, the matrix A can be transformed into the upper triangular matrix B using an elementary row transformation matrix $T_{r\times r}$. Besides, since all diagonal elements of A are 1 while all of the lower off-diagonal elements of A are negative, $T_{r\times r}$ is thus non-negative, which can be directly obtained from Gaussian elimination.

Furthermore, because $T_{r\times r}$ is a non-negative elementary row transformation matrix, and all elements of A in the upper diagonal are negative, $b_{ij} \leq 0$ holds for all b_{ij} in B.

2) Sufficiency. If $\varepsilon\delta^r < 1$, r-PTSI exists. For convenience, r-PTSI is denoted by $\widehat{X}^{(r)}$. Since $A\widehat{X}^{(r)} = N$ and $T_{r\times r}$ is an non-negative elementary row transformation matrix, $B\widehat{X}^{(r)} = TN > 0$. Further, since $\widehat{X}^{(r)} > 0$ and $b_{ij} \leq 0$ for all b_{ij} in B, $a_i > 0$ holds for all $i \in [1, r-1]$.

Necessity. If $a_i > 0$ for all $i \in [1, r-1]$, we can find a unique positive solution to equalities $BX = TN$ as follows.

Denote $TN = (c_1, c_2, \ldots, c_r)^T$. The unique solution $X = (X_r, X_{r-1}, \ldots, X_1)^T$ to the equalities $BX = TN$ is obviously $\left(\frac{c_r}{a_{r-1}}, \frac{c_{r-1}-X_r b_{(r-1)r}}{a_{r-2}}, \ldots, c_1 + \sum_{i=r}^{2}\gamma X_i\right)^T$. Since $X > 0$ and $AX = N$, $\varepsilon\delta^r < 1$ holds based on Lemma 2.

3) Because $T_{r\times r}$ is a non-negative elementary row transformation matrix, any vector $\overline{X}$ satisfying $A\overline{X} \geq N$ also satisfies $B\overline{X} \geq TN$. Besides, any vector $\overline{X}$ satisfying $A\overline{X} = N$ also satisfies $B\overline{X} = TN$.

Since $\widehat{X}^{(r)}$ satisfies the equations $B\widehat{X}^{(r)} = TN$, $B\left(\overline{X} - \widehat{X}^{(r)}\right) \geq 0$ for any vector $\overline{X}$ satisfying $A\overline{X} \geq N$. The last element of the vector $B\left(\overline{X} - \widehat{X}^{(r)}\right)$ is $a_{r-1}\left(\overline{X}_r - \widehat{X}_r^{(r)}\right)$, $\overline{X}_r \geq \widehat{X}_r^{(r)}$ thus holds because $a_{r-1} > 0$ and $B\left(\overline{X} - \widehat{X}^{(r)}\right) \geq 0$. Similarly, $\overline{X}_{r-1} \geq \widehat{X}_{r-1}^{(r)}$ also holds because $a_{r-1} > 0$, $a_{r-2} > 0$, $b_{(r-1)r} \leq 0$, and $B(\overline{X} - \widehat{X}^{(r)}) \geq 0$. Iteratively, since $a_i > 0$ for any $i \in [1, r-2]$, all $b_{ij} \leq 0$ for any $i \in [2, r-2]$ and $j \in [3, r]$, $\overline{X}_i \geq \widehat{X}_i^{(r)}$ for any $i \in [1, r-2]$. Therefore, $\overline{X} \geq \widehat{X}^{(r)}$.

4) Assume $\exists i \in [1, r-1]$ such that $a_i \leq 0$. Using a similar proof as 3, $p_i < 0$ must be true to ensure the existence of solutions to $AX \geq N$, which leads to contradictions. ■

C. Proof of Theorem 1

Proof. 1) Sufficiency. If $\varepsilon\delta^r < 1$, based on Lemma 3, $\sum_{i=1}^{r}\overline{X}_i \geq \sum_{i=1}^{r}\widehat{X}_i^{(r)}$ holds for any feasible solution $\left(\overline{X}_r, \overline{X}_{r-1}, \ldots, \overline{X}_1\right)$ to MPAr PT. Therefore, $\sum_{i=1}^{r}\frac{\widehat{X}_i^{(r)}}{G'_i} \leq \sum_{i=1}^{r}\frac{\overline{X}_i}{G'_i}$, where $\{G'_1, G'_2, \ldots, G'_r\}$ is any permutation of $\{G_1, G_2, \ldots, G_r\}$.

Since $\widehat{X}_r^{(r)} \geq \widehat{X}_{r-1}^{(r)} \geq \cdots \geq \widehat{X}_1^{(r)}$ and $G_r \geq G_{r-1} \geq \cdots \geq G_1$, based on the ordering inequality theorem, we know that $\sum_{i=1}^{r}\frac{\widehat{X}_i^{(r)}}{G_i} \leq \sum_{i=1}^{r}\frac{\widehat{X}_i^{(r)}}{G'_i}$.

Combining the above two inequalities together, we get $\sum_{i=1}^{r}\frac{\widehat{X}_i^{(r)}}{G_i} \leq \sum_{i=1}^{r}\frac{\overline{X}_i}{G'_i}$. Since $\left(\frac{\widehat{X}_r^{(r)}}{G_r}, \frac{\widehat{X}_{r-1}^{(r)}}{G_{r-1}}, \ldots, \frac{\widehat{X}_1^{(r)}}{G_1}\right)$ is a feasible solution to MPAr PT, and $\{\frac{\overline{X}_r}{G'_r}, \frac{\overline{X}_{r-1}}{G'_{r-1}}, \ldots, \frac{\overline{X}_1}{G'_1}\}$ represents any feasible solution to MPAr PT, $\left(\frac{\widehat{X}_r^{(r)}}{G_r}, \frac{\widehat{X}_{r-1}^{(r)}}{G_{r-1}}, \ldots, \frac{\widehat{X}_1^{(r)}}{G_1}\right)$ is thus the optimal solution to MPAr PT.

Necessity. If $\left(\frac{\widehat{X}_r^{(r)}}{G_r}, \frac{\widehat{X}_{r-1}^{(r)}}{G_{r-1}}, \ldots, \frac{\widehat{X}_1^{(r)}}{G_1}\right)$ is the optimal solution to MPAr PT, $A\widehat{X}^{(r)} \geq N$ holds, and $B\widehat{X}^{(r)} \geq TN > 0$.

We now prove the necessity by contradiction. Assume there is an $l \in [1, r-1]$ which satisfies $a_l \leq 0$, and $a_h \geq 0$ for all $h \in [l+1, r-1]$. Since for all b_{ij} in B, $b_{ij} \leq 0$ holds based on Lemma 3.1. On the other hand, to satisfy $B\widehat{X}^{(r)} > 0$, $\widehat{X}_h^{(r)} > 0$ for any $h \in [l+1, r]$ and $\widehat{X}_l^{(r)} < 0$ must hold simultaneously, which contradicts with the prerequisite $\widehat{X}^{(r)} > 0$.

2) It follows from Lemma 3.2 and Lemma 3.4. ■

D. Proof of Lemma 4

Proof.

satisfying $|S_2| \geq 2 + |S_1|$, where $|S_1|$ is the cardinality of S_1. If the user which is decoded first in S_2 is moved to S_1, a new power scheduling strategy will thus come into being. Based on Theorem 1, since $\widehat{X}_i^{(r)} < \widehat{X}_{i+1}^{(r)}$ for any $i \in [1, r-1]$, the aggregate power consumption of the new-formed scheduling strategy is less than that of the optimal one, which contradicts the optimality.

Similarly, there could not be a slot which includes more than $\lceil n/L \rceil$ UEs. Therefore, Lemma 4.1 is proved.

To prove Lemma 4.2, assume there are q slots each of which has $\lfloor n/L \rfloor$ users. Since $q\lfloor n/L \rfloor + (L-q)\lceil n/L \rceil = n$, $q = L\lceil n/L \rceil - n$ holds. ∎

E. Proof of Theorem 2

Proof. Sufficiency. 1) Based on the construction of $GH(n,k,L)$, and the mapping scheme that the edge (u_i, T_{hj}) means that u_i will be scheduled in slot h, any feasible UE scheduling strategy satisfying Lemma 4 can be mapped to a maximal matching of $GH(n,k,L)$, and vice versa. In other words, feasible UE scheduling strategies and the maximal matchings of $GH(n,k,L)$ have a one-to-one mapping.

2) For the edge (u_i, T_{hj}) in $GH(n,k,L)$, based on Theorem 1, $\frac{\widehat{x}_j}{G_i}$ is the minimal transmit power allocated to u_i if its decoding phase is j. Based on the above conclusions, for any maximal matching of $GH(n,k,L)$, its weighted sum is $nM - A$, where A is the minimum aggregate power consumption of all UEs for the corresponding scheduling strategy. So, the MWM of $GH(n,k,L)$ is the optimal solution to RMLPSI-k SIC, which is just the function of the tenth line in Algorithm 1.

Necessity. For any feasible scheduling strategy of the RMLPSI-k SIC problem, there must be a slot where there are at least $\lceil n/L \rceil$ UEs. For this slot, based on Theorem 1.2, there is no feasible power allocation strategy if $\varepsilon\delta^{\lceil n/L \rceil} \geq 1$. ∎

F. Proof of Theorem 3

Proof. Let $A = \{\widehat{X}_1^{(\lfloor n/L \rfloor)}, \widehat{X}_2^{(\lfloor n/L \rfloor)}, \ldots, \widehat{X}_{\lfloor n/L \rfloor}^{(\lfloor n/L \rfloor)}\}$ and $B = \{\widehat{X}_1^{(\lceil n/L \rceil)}, \widehat{X}_2^{(\lceil n/L \rceil)}, \ldots, \widehat{X}_{\lceil n/L \rceil}^{(\lceil n/L \rceil)}\}$. We construct a sequence which includes all elements of A for $L\lceil n/L \rceil - n$ times, and all elements of B for $L - L\lceil n/L \rceil + n$ times. Then, the sequence is sorted in ascending order. For convenience, denote the sorted sequence as $\langle c_1, c_2, \ldots, c_n \rangle$, and let $\langle b_1, b_2, \ldots, b_n \rangle = \langle \frac{1}{G_n}, \frac{1}{G_{n-1}}, \ldots, \frac{1}{G_1} \rangle$. Then, the output of Algorithm 2 is the optimal solution to the following problem:

$$\begin{aligned} &\min_{\{X_{ij}\}} \sum_{i,j=[1,n]} X_{ij} c_i b_j \\ \text{s.t.}\quad & X_{ij} \in \{0,1\} \\ & \sum_{i=[1,n]} X_{ij} = 1 \quad \forall j \in [1,n] \\ & \sum_{j=[1,n]} X_{ij} = 1 \quad \forall i \in [1,n]. \end{aligned} \tag{5}$$

Based on the ordering inequality theorem, the optimal solution to (5) is $\{X_{ij}\}$ where $X_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$. Since the output of Algorithm 2 is constructed to be consistent with the optimal value of $\{X_{ij}\}$, Algorithm 2 outputs an MWM of $GH(n,k,L)$.

G. Proof of Theorem 4

Proof. 1) WLOG, assume $u_1, u_2, \ldots, u_r$, where $r \leq k$, are scheduled simultaneously in one slot. Therefore, for

$$\forall l \in [2, r-1], \frac{G_l[tp_l^{(\rho\gamma)}]}{\sum_{i=1}^{l-1} G_i[tp_i^{(\rho\gamma)}] + \varepsilon \sum_{i=l+1}^{r} G_i[tp_i^{(\rho\gamma)}] + n_0}$$

$$\geq \frac{G_l[tp_l^{(\rho\gamma)}]}{\rho(\sum_{i=1}^{l-1} G_i tp_i^{(\rho\gamma)} + \varepsilon \sum_{i=l+1}^{r} G_i tp_i^{(\rho\gamma)} + n_0)} \geq \frac{\rho\gamma}{\rho}$$

$$= \gamma, \frac{G_1[tp_1^{(\rho\gamma)}]}{\sum_{i=2}^{r} G_i[tp_i^{(\rho\gamma)}] + n_0} \geq \frac{G_l[tp_l^{(\rho\gamma)}]}{\rho(\sum_{i=2}^{r} G_l tp_i^{(\rho\gamma)} + n_0)} \geq \gamma,$$

and $\frac{G_r[tp_r^{(\rho\gamma)}]}{\varepsilon\sum_{i=1}^{r-1} G_i[tp_i^{(\rho\gamma)}] + n_0} \geq \frac{G_r[tp_r^{(\rho\gamma)}]}{\rho(\varepsilon\sum_{i=1}^{r-1} G_i tp_i^{(\rho\gamma)} + n_0)} \geq \gamma$ because $tp_i^{(\rho\gamma)} \leq [tp_i^{(\rho\gamma)}] \leq \rho tp_i^{(\rho\gamma)}$ for $\forall i \in [1,r]$. Therefore, $([tp_1^{(\rho\gamma)}], [tp_2^{(\rho\gamma)}], \ldots, [tp_r^{(\rho\gamma)}])$ is a feasible solution to RMDLPSI-k SIC in the slot. Since the same proof is valid for other slots, Algorithm 3 outputs a feasible solution to RMDLPSI-k SIC.

2) WLOG, we still assume that $u_1, u_2, \ldots, u_r$, where $r \leq k$, are scheduled simultaneously in one slot. Since $tp_i^{(\rho\gamma)} \leq [tp_i^{(\rho\gamma)}] \leq \rho tp_i^{(\rho\gamma)}$ for $\forall i \in [1,r]$, $\sum_{i=1}^{r}[tp_i^{(\rho\gamma)}] \leq \rho\sum_{i=1}^{r} tp_i^{(\rho\gamma)}$.

Denoting the optimal solution to RMDLPSI-k SIC in the slot as $(\widehat{tp}_1^{(\gamma)}, \widehat{tp}_2^{(\gamma)}, \ldots, \widehat{tp}_r^{(\gamma)})$ and that to RMLPSI-k SIC as $(tp_1^{(\gamma)}, tp_2^{(\gamma)}, \ldots, tp_r^{(\gamma)})$, $\sum_{i=1}^{r} \widehat{tp}_i^{(\gamma)} \geq \sum_{i=1}^{r} tp_i^{(\gamma)}$ holds since $\widehat{tp}_i^{(\gamma)} \geq tp_i^{(r)}$ for $\forall i \in [1,r]$.

For $tp_i^{(\gamma)}$, $\forall i \in [1,r]$, if we allocate power $\rho^i tp_i^{(\gamma)}$ to u_i, $(\rho tp_1^{(\gamma)}, \rho^2 tp_2^{(\gamma)}, \ldots, \rho^r tp_r^{(\gamma)})$ is a feasible solution to RMLPSI-k SIC with the decoding threshold being $\rho\gamma$. Therefore

$$\sum_{i=1}^{r} tp_i^{(\rho\gamma)} \leq \sum_{i=1}^{r} \rho^i tp_i^{(\gamma)} \leq \rho^r \sum_{i=1}^{r} tp_i^{(\gamma)}. \tag{6}$$

Since (6) is always satisfied for any value of r where $r \leq \lceil n/L \rceil$, $\frac{\sum_{i=1}^{n} tp_i^{(\rho\gamma)}}{\sum_{i=1}^{n} tp_i^{(\gamma)}} \leq \rho^{\lceil n/L \rceil}$. Therefore, $\frac{\sum_{i=1}^{n}[tp_i^{(\rho\gamma)}]}{\sum_{i=1}^{n} \widehat{tp}_i^{(\gamma)}} \leq \rho^{(\lceil n/L \rceil + 1)}$. ∎

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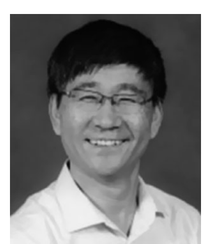

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