

# Multi-view classifier based on Probabilistic Collaborative Representation and Latent Representation

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## Abstract:

This paper proposes a multi-view classifier based on Probabilistic Collaborative Representation and Latent Representation (PCRLRL). The proposed method first introduces a probabilistic collaborative representation model to capture the relationship between different views. Then, a latent representation model is used to extract the shared information across views. Finally, a multi-view classifier is designed to combine the information from both views for classification. The proposed method is evaluated on several datasets and shows superior performance compared to other methods.

## Key Words:

Multi-view classification, Probabilistic Collaborative Representation, Latent Representation

## 1 Introduction

In recent years, multi-view learning has attracted much attention due to its wide application in many fields. Multi-view learning aims to exploit the complementary information from multiple views to improve the performance of machine learning tasks. There are many methods for multi-view learning, such as multi-view principal component analysis (MVA), multi-view kernel learning (MKL), and multi-view support vector machines (MVSVM). However, these methods often ignore the relationship between different views and the shared information across views. In this paper, we propose a multi-view classifier based on Probabilistic Collaborative Representation and Latent Representation (PCRLRL). The proposed method first introduces a probabilistic collaborative representation model to capture the relationship between different views. Then, a latent representation model is used to extract the shared information across views. Finally, a multi-view classifier is designed to combine the information from both views for classification. The proposed method is evaluated on several datasets and shows superior performance compared to other methods.

The proposed method is based on the idea of probabilistic collaborative representation and latent representation. In the probabilistic collaborative representation model, the relationship between different views is captured by a probabilistic model. In the latent representation model, the shared information across views is extracted by a latent representation model. Finally, a multi-view classifier is designed to combine the information from both views for classification. The proposed method is evaluated on several datasets and shows superior performance compared to other methods.

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$X \sim \mathcal{N}(\mu, \sigma^2)$ ,  $Y \sim \mathcal{N}(\mu, \sigma^2)$  independent

$$P(l(y) \in l_X) = P(l(y) = l(x) | l(x) \in l_X) \cdot P(l(x) \in l_X)$$

$$P(l(y) = l(x) | l(x) \in l_X) = P(y = x | x \in l_X)$$

$$\begin{bmatrix} y_{1,n_y} \\ y_{2,n_y} \\ \vdots \\ y_{m,n_y} \\ \vdots \\ y_{M,n_y} \end{bmatrix} = [x_1, x_2, \dots, x_{n_X}, \dots, x_{N_X}] \begin{bmatrix} \hat{\alpha}_{1,n_y} \\ \hat{\alpha}_{2,n_y} \\ \vdots \\ \hat{\alpha}_{n_X,n_y} \\ \vdots \\ \hat{\alpha}_{N_X,n_y} \end{bmatrix} \quad (15)$$

$$= \hat{\alpha}_{1,n_y} \begin{bmatrix} x_{1,1} \\ x_{2,1} \\ \vdots \\ x_{m,1} \\ \vdots \\ x_{M,1} \end{bmatrix} + \dots + \hat{\alpha}_{n_X,n_y} \begin{bmatrix} x_{1,n_X} \\ x_{2,n_X} \\ \vdots \\ x_{m,n_X} \\ \vdots \\ x_{M,n_X} \end{bmatrix} + \dots + \hat{\alpha}_{N_X,n_y} \begin{bmatrix} x_{1,N_X} \\ x_{2,N_X} \\ \vdots \\ x_{m,N_X} \\ \vdots \\ x_{M,N_X} \end{bmatrix}$$

The input-output relationship of the system can be expressed as follows:

$$\min_Z L(X, XZ) + \lambda \|Z\| \quad (16)$$

where  $L(\cdot)$  is the loss function,  $\lambda > 0$  is the regularization parameter, and  $Z$  is the vector of coefficients.

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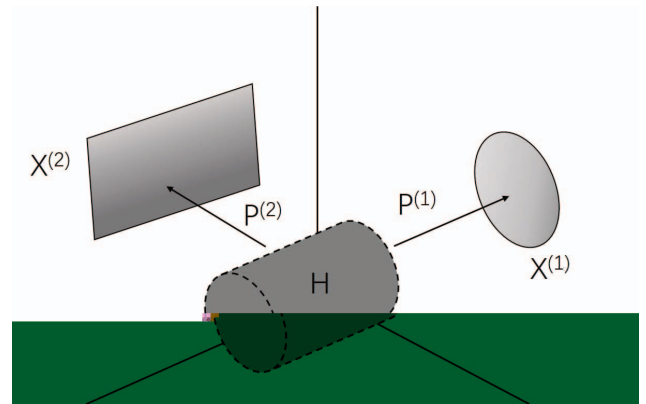


Figure 1: A 3D diagram illustrating the input-output relationship of the system.

The input-output relationship of the system can be expressed as follows:

$$x_i^{(v)} = P^{(v)} h_i + e_i^{(v)} \quad (1)$$

where  $e_i^{(v)}$  is the noise term.

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$$\min_{P,H,Z,E_V,E_S} \|E_V\|_{2,1} + \lambda_1 \|E_S\|_{2,1} + \lambda_2 \|Z\|_2 \quad (21)$$

$$s.t. X = PH + E_V, H = HZ + E_S, PP^T = I$$

$$L(P, H, Z, E_V, E_S, J)$$

$$= \|E\|_{2,1} + \lambda \|J\|_2 + (W_1, X - PH - E_V) + (W_2, H - HZ - E_S) + (W_3, J - Z) \quad (22)$$

$$s.t. E = [E_V; E_S]; PP^T = I$$

$$P^* = \arg \min_P (W_1, X - PH - E_V) \quad (23)$$

$$s.t. PP^T = I$$

$$\min_R \|Q - GR\|_F^2, s.t. R^T R = I, R = UV^T, U, V \in \mathbb{R}^{n \times n}, G^T Q = 0 \quad (24)$$

$$(P^*)^T = UV^T, U, V \in \mathbb{R}^{n \times n}, G^T Q = 0 \quad (25)$$

$$H^* = \arg \min_H (W_1, X - PH - E_V) + (W_2, H - HZ - E_S) \quad (26)$$

$$AH + HB = C \quad (27)$$

$$A = \mu P^T P \quad (28)$$

$$B = \mu(ZZ^T - Z - Z^T + I) \quad (28)$$

$$C = (P^T W_1 + W_2(Z^T - I)) + \mu(P^T X + E_S^T - P^T E_V - E_S Z^T) \quad (29)$$

$$Z^* = \arg \min_Z (W_3, J - Z) + (W_2, H - HZ - E_S) \quad (30)$$

$$Z^* = (H^T H + I)^{-1} [(J + H^T H - H^T E_S) + (W_3 + H^T W_2)/\mu] \quad (31)$$

$$E^* = \arg \min_E \|E\|_{2,1} + (W_1, X - PH - E_V) + (W_2, H - HZ - E_S) = \arg \min_E \frac{1}{\mu} \|E\|_{2,1} + \frac{1}{2} \|E - G\|_F^2 \quad (32)$$

$$J^* = \arg \min_J \lambda \|J\|_* + (W_3, J - Z) = \frac{\lambda}{\mu} \|J\|_* + \frac{1}{2} \|J - (Z - W_3/\mu)\|_F^2 \quad (33)$$

$$W_1 = W_1 + \mu(X - PH - E_V) \quad (34)$$

$$W_2 = W_2 + \mu(H - HZ - E_S)$$

$$W_3 = W_3 + \mu(J - Z)$$

$$X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^N, \lambda = \rho \mu, \mu = \min(\rho \mu; \max_\mu) \quad (35)$$

$$\|X - PH - E_V\|_\infty < \varepsilon, \|H - PH - E_S\|_\infty < \varepsilon, \|J - Z\|_\infty < \varepsilon \quad (36)$$

$$P, H, Z, E \quad (37)$$

$$H, H \quad (38)$$

$$f_i, h \quad (39)$$

$$H, H \quad (40)$$

$$f_i, h \quad (41)$$

$$f_i, h \quad (42)$$

### Algorithm 1

**Input:**  $X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^N, \lambda, \rho, \mu, \varepsilon$

- $P = 0, E_V = 0, E_S = 0, J = Z = 0, W_1 = 0, W_2 = 0, W_3 = 0, \mu = 10^{-6}, \rho = 1.2, \varepsilon = 10^{-4}, \max_\mu = 10^6, H = 0$
- repeat**
- $P, H, Z, E_V, E_S, J$
- $W_1, W_2, W_3$
- $\mu = \min(\rho \mu; \max_\mu)$
- $\|X - PH - E_V\|_\infty < \varepsilon, \|H - PH - E_S\|_\infty < \varepsilon, \|J - Z\|_\infty < \varepsilon$
- until**

**Output:**  $P, H, Z, E$

### 3 Experimental results

#### 3.1 ProCRC-MV

$X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^{N_X}$   
 $X^{M \times N_X} = [x_1, \dots, x_{n_X}, \dots, x_{N_X}], n_X \in \{1, \dots, N_X\}$ .  
 $Y = \{[y_i^1, \dots, y_i^V]\}_{i=1}^{N_Y}$   
 $Y^{M \times N_Y} = [y_1, \dots, y_{n_Y}, \dots, y_{N_Y}], n_Y \in \{1, \dots, N_Y\}$ .  
 $M = \{l_i\}_{i=1}^{N_X}, Y^{M \times N_Y}$

	1	h		
256	2	2016		8
96	3	64		6
400	3	3304 4096 6 50		40
685	4	4633 4659 4684 4665		5
195	2	585 1 03		5
18	2	561 1 03		5
265	2	95 1 03		5

15, 10,  
 0.8 0.2  
 0.6 0.2 0.2  
 2  
 3.

$X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^{N_X}$   
 $X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^{N_X}$   
 $Y = \{[y_i^1, \dots, y_i^V]\}_{i=1}^{N_Y}$   
 $H$   
 $0.002$   
 $2$   
 $10$

#### 3.2 LProCRC-MV

$X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^{N_X}$   
 $X = \{[x_i^1, \dots, x_i^V]\}_{i=1}^{N_X}$   
 $Y = \{[y_i^1, \dots, y_i^V]\}_{i=1}^{N_Y}$   
 $H$   
 $4$  5  
 $4$  5  
 $1$   
 $h$

2. Experimental Results of the Proposed Scheme

Method	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7
Proposed	88.9±3.4	91.6±2.3	92.6±5.4	90.4±2.0	2.5±5.5	6.1±5.4	85.0±4.1
Reference [10]	63.8±4.0	95.0±0.2	89.2±1.9	2.8±0.5	49.5±1.4	69.2±4.3	51.4±3.8
Reference [11]	50.1±2.6	95.4±0.1	5.5±0	3.8±1.2	41.4±1.1	58.0±1.1	53.0±1.5
Reference [12]	33.5±1.0	100±0	81.8±1.9	95.5±2.8	60.1±5.1	65.1±5.0	64.5±2.5
Reference [13]	98.6±1.3	100±0	98.6±1.3	96.1±1.1	9.5±6.4	0.3±6.1	8.5±5.0

3. Experimental Results of the Proposed Scheme with Scenario 1

Method	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7
Proposed	81.9±3.8	91.9±2.6	92.65±4.2	89.1±2.3	59.4±8.1	61.8±11.5	69.4±9.1
Reference [10]	62.5±4.1	93.9±1.3	81.0±3.0	1.2±0.1	32.9±6.2	49.1±5.0	31.8±3.1
Reference [11]	50.9±2.5	96.1±0.1	55.5±0	32.3±3.2	26.1±1.5	51.1±1.4	40.0±2.5
Reference [12]	32.2±5.9	100±0	84.2±2.2	93.4±6.4	42.8±4.6	53.6±8.5	0.8±2.1
Reference [13]	98.1±1.2	100±0	98.1±1.2	96.0±2.0	66.0±8.4	60.3±1.5	66.0±8.4

4. Experimental Results of the Proposed Scheme with Scenario 4

Method	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7
Proposed	88.9±3.4	91.6±2.3	92.6±5.4	90.4±2.0	2.5±5.5	6.1±5.4	85.0±4.1
Reference [10]	63.8±4.0	95.0±0.2	89.2±1.9	2.8±0.5	49.5±1.4	69.2±4.3	51.4±3.8
Reference [11]	50.1±2.6	95.4±0.1	5.5±0	3.8±1.2	41.4±1.1	58.0±1.1	53.0±1.5
Reference [12]	33.5±1.0	100±0	81.8±1.9	95.5±2.8	60.1±5.1	65.1±5.0	64.5±2.5
Reference [13]	98.6±1.3	100±0	98.6±1.3	96.1±1.1	9.5±6.4	0.3±6.1	8.5±5.0
Reference [14]	95.5±1.0	100±0	98.8±0.5	94.9±1.5	80.8±4.4	9.6±5.1	86.8±4.8

5. Experimental Results of the Proposed Scheme with Scenario 1 and Scenario 4

Method	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7
Proposed	81.9±3.8	91.9±2.6	92.65±4.2	89.1±2.3	59.4±8.1	61.8±11.5	69.4±9.1
Reference [10]	62.5±4.1	93.9±1.3	81.0±3.0	1.2±0.1	32.9±6.2	49.1±5.0	31.8±3.1
Reference [11]	50.9±2.5	96.1±0.1	55.5±0	32.3±3.2	26.1±1.5	51.1±1.4	40.0±2.5
Reference [12]	32.2±5.9	100±0	84.2±2.2	93.4±6.4	42.8±4.6	53.6±8.5	50.8±2.1
Reference [13]	98.1±1.2	100±0	98.1±1.2	96.0±2.0	66.0±8.4	60.3±1.5	66.0±8.4
Reference [14]	95.4±1.1	100±0	98.0±0.8	95.0±0.3	69.6±6.1	65.5±4.9	63.8±6.3

Proposed scheme, the proposed scheme with scenario 1, the proposed scheme with scenario 4, and the proposed scheme with scenario 1 and scenario 4 are compared with the reference schemes. The results show that the proposed scheme has the best performance in terms of the success rate of the proposed scheme. The success rate of the proposed scheme is 88.9% in scenario 1, 91.6% in scenario 2, 92.6% in scenario 3, 90.4% in scenario 4, 2.5% in scenario 5, 6.1% in scenario 6, and 85.0% in scenario 7. The success rate of the proposed scheme with scenario 1 is 81.9% in scenario 1, 91.9% in scenario 2, 92.65% in scenario 3, 89.1% in scenario 4, 59.4% in scenario 5, 61.8% in scenario 6, and 69.4% in scenario 7. The success rate of the proposed scheme with scenario 4 is 88.9% in scenario 1, 91.6% in scenario 2, 92.6% in scenario 3, 90.4% in scenario 4, 2.5% in scenario 5, 6.1% in scenario 6, and 85.0% in scenario 7. The success rate of the proposed scheme with scenario 1 and scenario 4 is 81.9% in scenario 1, 91.9% in scenario 2, 92.65% in scenario 3, 89.1% in scenario 4, 59.4% in scenario 5, 61.8% in scenario 6, and 69.4% in scenario 7. The success rate of the proposed scheme is higher than the reference schemes in all scenarios. The success rate of the proposed scheme is 88.9% in scenario 1, 91.6% in scenario 2, 92.6% in scenario 3, 90.4% in scenario 4, 2.5% in scenario 5, 6.1% in scenario 6, and 85.0% in scenario 7. The success rate of the proposed scheme with scenario 1 is 81.9% in scenario 1, 91.9% in scenario 2, 92.65% in scenario 3, 89.1% in scenario 4, 59.4% in scenario 5, 61.8% in scenario 6, and 69.4% in scenario 7. The success rate of the proposed scheme with scenario 4 is 88.9% in scenario 1, 91.6% in scenario 2, 92.6% in scenario 3, 90.4% in scenario 4, 2.5% in scenario 5, 6.1% in scenario 6, and 85.0% in scenario 7. The success rate of the proposed scheme with scenario 1 and scenario 4 is 81.9% in scenario 1, 91.9% in scenario 2, 92.65% in scenario 3, 89.1% in scenario 4, 59.4% in scenario 5, 61.8% in scenario 6, and 69.4% in scenario 7.

#### 4 Conclusions and Discussion

The proposed scheme is compared with the reference schemes in terms of the success rate of the proposed scheme. The results show that the proposed scheme has the best performance in terms of the success rate of the proposed scheme. The success rate of the proposed scheme is 88.9% in scenario 1, 91.6% in scenario 2, 92.6% in scenario 3, 90.4% in scenario 4, 2.5% in scenario 5, 6.1% in scenario 6, and 85.0% in scenario 7. The success rate of the proposed scheme with scenario 1 is 81.9% in scenario 1, 91.9% in scenario 2, 92.65% in scenario 3, 89.1% in scenario 4, 59.4% in scenario 5, 61.8% in scenario 6, and 69.4% in scenario 7. The success rate of the proposed scheme with scenario 4 is 88.9% in scenario 1, 91.6% in scenario 2, 92.6% in scenario 3, 90.4% in scenario 4, 2.5% in scenario 5, 6.1% in scenario 6, and 85.0% in scenario 7. The success rate of the proposed scheme with scenario 1 and scenario 4 is 81.9% in scenario 1, 91.9% in scenario 2, 92.65% in scenario 3, 89.1% in scenario 4, 59.4% in scenario 5, 61.8% in scenario 6, and 69.4% in scenario 7.

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