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J -J L , -L L *, E

Department of Automation, China University of Petroleum Beijing, 102249, China

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F . T . T . B . I , T . T 2020 T I C E . P E B. . A .

1. Introduction

I fi (PSE), P fi 1,2 . D fi . P , . I , . F , fi . I (MIDO) 3–6 ; fl

7 . I FCCU CO fl CO . M . T FCCU CO 8 fl 2 . T

. I P 20 B 21 fi fi . I 22 . A

(NLP). A . M 23,24 , 25,26 27,28 . B NLP, : 1) ; 2) ; 3) . A . N fl T fi 29,30 fi fi . T (NCO-) 31, 32 , 7 . NSGBD (C P) 33 . I . I NSGBD NSGBD. I NCO- T T FCCU : I § 2, fi

CO fl . I § 3.1, § 3.2 § 3.3. I § 4, FCCU § 3.3.

2. Batch properties of FCCU

T FCCU fi 7,34–36 . A FCCU fi F . 1. H , T fi fi - . T fi fl . T . T fl : -fl -fl . T : 1) ; 2) ; 3) fl . M fl T FCCU PID . I fi , PID FCCU fi fl . H A , FCCU fl CO . T fi fl . C fl 37 , fl . T fl

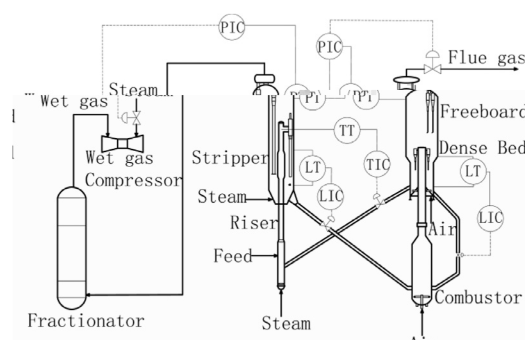


Fig. 1. S FCCU fi .

. T . H , fl fl . M FCCU CO CO CO_2 . T FCCU CO . I , CO . I FCCU, CO . I FCCU, , CO - . A , . CO . O 8 , 2 .

3. Hybrid parametric dynamic optimization

3.1. Mathematical formulation of hybrid parametric dynamic optimization

T . F , (DAE):

$$\dot{x}_d = f_d\left(t, x_d(t), x_a(t), u(t), \bar{u}\right) \quad (1\ )$$

$$0 = f_a\left(t, x_d(t), x_a(t), u(t), \bar{u}\right) \quad (1\ )$$

$x_d(t) \in R^{n_d} / x_a(t) \in R^{n_a}$ / ; $u(t)/\bar{u}$ / . I $x_d(t_0)$, fi . O , $\bar{u}$.

T ,

$$J(u(t), \bar{u}) = \int_{t_0}^{t_f} \left(-r(t, x_d(t), x_a(t), u, \bar{u}) + c_1(t, x_d(t), x_a(t), u, \bar{u})\right) dt + c_2(\bar{u}) \quad (2)$$

J , $r\left(t, x_d(t), x_a(t), u, \bar{u}\right)$ $c_1\left(t, x_d(t), x_a(t), u, \bar{u}\right)$ $c_2(\bar{u})$ t_0. A $\tilde{x}(t)$ - J. I ,

$$\tilde{x} = -r + c_1 \quad (3)$$

$$\tilde{x}(t_0) = 0 \quad (4)$$

, $x_d(t)$. T J fi :

$$J\left(x(t_f), \bar{u}\right) = \tilde{x}(t_f) + c_2(\bar{u}) \quad (5)$$

$x(t) = (x_d(t)^T, x_a(t)^T)^T$.

T

$$x^{lb} \le x(t) \le x^{ub} \quad (6\ )$$

$$u^{lb} \le u(t) \le u^{ub} \quad (6\ )$$

$x^{lb} = ((x_d^{lb})^T, (x_a^{lb})^T)^T$, $x^{ub} = ((x_d^{ub})^T, (x_a^{ub})^T)^T$, x_d^{lb} x_d^{ub} ; x_a^{lb} x_a^{ub} ; u^{lb} u^{ub} . M , $\bar{u}$

$$\bar{u}^{lb} \le \bar{u} \le \bar{u}^{ub} \quad (7)$$

$\bar{u}^{lb}$ $\bar{u}^{ub}$. T -

$$x^{flb} \le x(t_f) \le x^{fub} \quad (8)$$

$x^{flb} = ((x_d^{flb})^T, (x_a^{flb})^T)^T$, $x^{fub} = ((x_d^{fub})^T, (x_a^{fub})^T)^T$, x_d^{flb} x_d^{fub} - ; x_a^{flb} x_a^{fub} - ; T - - . F , $u(t)$ - $\bar{u}$ E . (5), fi E . (1), (6), (7) (8). T P (P1).

Problem (P1):

$$J\left(x(t_f), \bar{u}\right) \quad (9\ )$$

$$. .\ x_d(t_0) = x_d^0 \quad (9\ )$$

$$\dot{x}_d = f_d\left(t, x_d(t), x_a(t), u(t), \bar{u}\right) \quad (9\ )$$

$$0 = f_a\left(t, x_d(t), x_a(t), u(t), \bar{u}\right) \quad (9\ )$$

$$G_{\bar{u}}(\bar{u}) \le 0 \quad (9\ )$$

$$G_u\left(u(t)\right) \le 0 \quad (9\ )$$

$$G_p\left(x(t)\right) \le 0 \quad (9\ )$$

$$G_e\left(x(t_f)\right) \le 0 \quad (9\ )$$

$$G_{\bar{u}}(\bar{u}) = \begin{pmatrix} \bar{u} - \bar{u}^{ub} \\ \bar{u}^{lb} - \bar{u} \end{pmatrix} \quad (9\ )$$

$$G_u\left(u(t)\right) = \begin{pmatrix} u(t) - u^{ub} \\ u^{lb} - u(t) \end{pmatrix} \quad (9\ )$$

$$G_p\left(x(t)\right) = \begin{pmatrix} x(t) - x^{ub} \\ x^{lb} - x(t) \end{pmatrix} \quad (9\ )$$

$$G_e\left(x(t_f)\right) = \begin{pmatrix} x(t_f) - x^{fub} \\ x^{flb} - x(t_f) \end{pmatrix} \quad (9\ )$$

A - , . I , fi - fi - NLP . F , fi . S fi ,

$\boldsymbol{u}(t) \in D,$

$$\boldsymbol{u}(t) \approx \widehat{\boldsymbol{u}}^N(t) = \sum_{i=1}^{N} \boldsymbol{u}_i^N \phi_i^N(t) \quad (10)$$

$\phi_i^N(t)$ $D^N = \{\phi_1^N(t), \phi_2^N(t), ..., \phi_N^N(t)\},$ $\boldsymbol{u}_i^N$ $\boldsymbol{u}(t)$ $\phi_i^N(t)$. U , L $\phi_i^N(t),$ $\left(\phi_i^N(t) \cap \phi_j^N(t)\right) = 0$ ( ) $\forall i \neq j$. H , $< \phi_i^N(t), \phi_j^N(t) > = 0$ $\forall i \neq j$ D_N. F - , t_0, t_f $N \geq 1$, . $t_0 < t_1 < t_2 < ... < t_N = t_f$, - $\phi_i^N(t)$

$$\phi_i^N(t) = \begin{cases} 1, t_{i-1} \leq t \leq t_i \\ 0 \end{cases} \quad (11)$$

B , $\boldsymbol{u}(t)$ $\widehat{\boldsymbol{u}}_N = ((\boldsymbol{u}_1^N)^T, ..., (\boldsymbol{u}_N^N)^T)^T$ - NLP. S - , - . F , $y(t)$ y^{ub} y^{lb}, $y^l(t)$ $y^u(t)$ $\dot{y}^l = (0, y^{lb} - y(t))$ $\dot{y}^u = (0, y(t) - y^{ub})$, $y^l(t_0) = 0$ $y^u(t_0) = 0$, - : $y^l(t_f) = 0$ $y^u(t_f) = 0$. M , $\boldsymbol{u}(t)$, $\boldsymbol{u}(t)$ $\widehat{\boldsymbol{u}}_N$. A , NLP P (P2).

Problem (P2):

$$_{\widehat{u}_N, \overline{u}} J\left(\boldsymbol{x}(t_f), \overline{\boldsymbol{u}}\right) \quad (12\ )$$

$$. . \boldsymbol{MX}(t_0) = \boldsymbol{X}_0 \quad (12\ )$$

$$\boldsymbol{M}\dot{\boldsymbol{X}}(t) = \boldsymbol{F}\left(t, \boldsymbol{X}(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}\right) \quad (12\ )$$

$$\boldsymbol{G}\left(\overline{\boldsymbol{u}}, \widehat{\boldsymbol{u}}_N, \boldsymbol{X}(t_f)\right) \leq \boldsymbol{0} \quad (12\ )$$

$$M = (\underbrace{1, ..., 1}_{3n_d + 2n_a}, \underbrace{0, ..., 0}_{n_a}) \quad (12\ )$$

$$\boldsymbol{X}(t) = \begin{pmatrix} \boldsymbol{x}_d(t) \\ \boldsymbol{x}^b(t) \\ \boldsymbol{x}_a(t) \end{pmatrix} \quad (12\ )$$

$$\boldsymbol{X}_0 = \begin{pmatrix} \boldsymbol{x}_d^0 \\ \boldsymbol{0} \\ \boldsymbol{0} \end{pmatrix} \quad (12\ )$$

$$\boldsymbol{F}\left(t, \boldsymbol{X}(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}\right) = \begin{pmatrix} \boldsymbol{f}_d\left(t, \boldsymbol{x}_d(t), \boldsymbol{x}_a(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}^j\right) \\ \boldsymbol{f}_b\left(t, \boldsymbol{x}(t)\right) \\ \boldsymbol{f}_a\left(t, \boldsymbol{x}_d(t), \boldsymbol{x}_a(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}^j\right) \end{pmatrix} \quad (12\ )$$

$$\boldsymbol{f}_b\left(t, \boldsymbol{x}(t)\right) = \begin{pmatrix} \left(\boldsymbol{0}, \boldsymbol{x}^{lb} - \boldsymbol{x}(t)\right) \\ \left(\boldsymbol{0}, \boldsymbol{x}(t) - \boldsymbol{x}^{ub}\right) \end{pmatrix} \quad (12\ )$$

$$\boldsymbol{G}\left(\overline{\boldsymbol{u}}, \widehat{\boldsymbol{u}}_N, \boldsymbol{X}(t_f)\right) = \begin{pmatrix} \boldsymbol{G}_{\overline{u}}(\overline{\boldsymbol{u}}) \\ \boldsymbol{G}'_u(\widehat{\boldsymbol{u}}_N) \\ \boldsymbol{G}'_e\left(\boldsymbol{x}^b(t_f)\right) \\ \boldsymbol{G}_e\left(\boldsymbol{x}(t_f)\right) \end{pmatrix} \quad (12\ )$$

$$\boldsymbol{G}'_u(\widehat{\boldsymbol{u}}_N) = \begin{pmatrix} \boldsymbol{G}_u(\boldsymbol{u}_1^N) \\ \vdots \\ \boldsymbol{G}_u(\boldsymbol{u}_N^N) \end{pmatrix} \quad (12\ )$$

$$\boldsymbol{G}'_e\left(\boldsymbol{x}^b(t_f)\right) = \boldsymbol{x}^b(t_f) \quad (12\ )$$

3.2. Nonconvex sensitivity-based generalized Benders decomposition

S , GBD- . R NSGBD C P 33 . I , - : (1) - (LP) B , , ; (2) - ; (3) B .

B $\overline{u}$ $\underline{u}$

$$\boldsymbol{h}(\overline{\boldsymbol{u}}, \underline{\boldsymbol{u}}) = \underline{\boldsymbol{u}} - \overline{\boldsymbol{u}} = \boldsymbol{0} \quad (13)$$

NSGBD P (P2) :

Problem (P3):

$$(\widehat{\boldsymbol{u}}_{N,a}^j | J_a^j) = {}_{\widehat{u}_N} J\left(\boldsymbol{x}(t_f), \overline{\boldsymbol{u}}_a^j\right) \quad (14\ )$$

$$. . \boldsymbol{MX}(t_0) = \boldsymbol{X}_0 \quad (14\ )$$

$$\boldsymbol{M}\dot{\boldsymbol{X}}(t) = \boldsymbol{F}\left(t, \boldsymbol{X}(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}^j\right) \quad (14\ )$$

$$\boldsymbol{G}\left(\overline{\boldsymbol{u}}_a^j, \widehat{\boldsymbol{u}}_N, \boldsymbol{X}(t_f)\right) \leq \boldsymbol{0} \quad (14\ )$$

J_a^j .

Problem (P4):

$$(\widehat{\boldsymbol{u}}_{N,b}^j, \overline{\boldsymbol{u}}_b^j | \boldsymbol{\mu}_a^j) = {}_{\widehat{u}_N, \overline{u}} J\left(\boldsymbol{x}(t_f), \overline{\boldsymbol{u}}\right) \quad (15\ )$$

$$. . \boldsymbol{MX}(t_0) = \boldsymbol{X}_0 \quad (15\ )$$

$$\boldsymbol{M}\dot{\boldsymbol{X}}(t) = \boldsymbol{F}\left(t, \boldsymbol{X}(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}\right) \quad (15\ )$$

$$\boldsymbol{G}\left(\overline{\boldsymbol{u}}, \widehat{\boldsymbol{u}}_N, \boldsymbol{X}(t_f)\right) \leq \boldsymbol{0} \quad (15\ )$$

$$\boldsymbol{h}\left(\overline{\boldsymbol{u}}, \underline{\boldsymbol{u}}_a^j\right) = \boldsymbol{0} \quad (15\ )$$

$\boldsymbol{\mu}_a^j$ (15).

Problem (P5):

$$(\widehat{\boldsymbol{u}}^{j'}_{N,c}, \overline{\boldsymbol{u}}^{j'}_{c}) = \quad {}_{\widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}} \parallel \overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j}_{t} \parallel^2_A \quad (16\ )$$

$$. \ . \ M\boldsymbol{X}(t_0) = \boldsymbol{X}_0 \quad (16\ )$$

$$M\dot{\boldsymbol{X}}(t) = \boldsymbol{F}\Big(t, \boldsymbol{X}(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}\Big) \quad (16\ )$$

$$\boldsymbol{G}(\overline{\boldsymbol{u}}, \widehat{\boldsymbol{u}}_N, \boldsymbol{X}(t_f)\Big) \le \boldsymbol{0} \quad (16\ )$$

$$\parallel \overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j} \parallel^2_A = (\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j})^T A (\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j}) \quad (16\ )$$

A fi .

Problem (P6):

$$(\alpha^{j'}_{d}, \widehat{\boldsymbol{u}}^{j'}_{N,d}, \overline{\boldsymbol{u}}^{j'}_{d} | \boldsymbol{\mu}^{j'}_{b}) = \quad {}_{\alpha, \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}} \alpha \quad (17\ )$$

$$. \ . \ M\boldsymbol{X}(t_0) = \boldsymbol{X}_0 \quad (17\ )$$

$$M\dot{\boldsymbol{X}}(t) = \boldsymbol{F}\Big(t, \boldsymbol{X}(t), \widehat{\boldsymbol{u}}_N, \overline{\boldsymbol{u}}\Big) \quad (17\ )$$

$$\boldsymbol{G}(\overline{\boldsymbol{u}}, \widehat{\boldsymbol{u}}_N, \boldsymbol{X}(t_f)\Big) - \alpha \boldsymbol{e} \le \boldsymbol{0} \quad (17\ )$$

$$\boldsymbol{h}(\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}^{j'}_{c}) = \boldsymbol{0} \quad (17\ )$$

$\boldsymbol{\mu}^{j'}_{b}$ (17) $\boldsymbol{e}$.
T $\overline{\boldsymbol{u}}$ (E . (7)) :

$$\boldsymbol{\mu}^{i}_{c} \cdot \Big(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{ub}\Big) \le 0, i \in K_u \quad (18\ )$$

$$\boldsymbol{\mu}^{i'}_{d} \cdot \Big(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{lb}\Big) \le 0, i' \in K_l \quad (18\ )$$

$\boldsymbol{\mu}^{i}_{c} = (\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0 \ldots, 0}_{l-i})$ $\boldsymbol{\mu}^{i'}_{d} = (\underbrace{0, \ldots, 0}_{i'-1}, -1, \underbrace{0 \ldots, 0}_{l-i'})$.

Problem (P7):

$$\Big(\eta^{k}_{b}, \underline{\boldsymbol{u}}^{k}_{b} | \boldsymbol{v}^{k}_{a}, \boldsymbol{v}^{k}_{b}, \boldsymbol{v}^{k}_{c}, \boldsymbol{v}^{k}_{d}\Big) = \quad {}_{\eta, \underline{\boldsymbol{u}}} \eta \quad (19\ )$$

$$\eta \ge J^{j}_{a} + \boldsymbol{\mu}^{j}_{a} \cdot \Big(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j}_{a}\Big), j \in K_{feas} \quad (19\ )$$

$$\boldsymbol{\mu}^{j'}_{b} \cdot \Big(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j'}_{c}\Big) \le 0, j' \in K_{infeas} \quad (19\ )$$

$$\boldsymbol{\mu}^{i}_{c} \cdot \Big(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{ub}\Big) \le 0, i \in K_u \quad (19\ )$$

$$\boldsymbol{\mu}^{i'}_{d} \cdot \Big(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{lb}\Big) \le 0, i' \in K_l \quad (19\ )$$

$\boldsymbol{v}^{k}_{a}$, $\boldsymbol{v}^{k}_{b}$, $\boldsymbol{v}^{k}_{c}$ $\boldsymbol{v}^{k}_{d}$ L K_{feas}, K_{infeas}, K_u K_l,
D fi

$$\Xi_a = \{j \in \ : (\boldsymbol{v}^{k}_{a})_j \ne 0\} \quad (20\ )$$

$$\Xi_b = \{j' \in \ : (\boldsymbol{v}^{k}_{b})_{j'} \ne 0\} \quad (20\ )$$

$$\Xi_c = \{i \in \ : (\boldsymbol{v}^{k}_{c})_i \ne 0\} \quad (20\ )$$

$$\Xi_d = \{i' \in \ : (\boldsymbol{v}^{k}_{d})_{i'} \ne 0\} \quad (20\ )$$

Problem (P8):

$$(\underline{\boldsymbol{u}}^{k}_{c}) = \quad {}_{\underline{\boldsymbol{u}}} \boldsymbol{\mu}^{k}_{a} \cdot \underline{\boldsymbol{u}} \quad (21\ )$$

$$\boldsymbol{\mu}^{j'}_{b} \cdot (\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{j'}_{c}) \le 0, j' \in K_{infeas} \quad (21\ )$$

$$\boldsymbol{\mu}^{i}_{c} \cdot (\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_{u}) \le 0, i \in K_u \quad (21\ )$$

$$\boldsymbol{\mu}^{i'}_{d} \cdot (\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_{l}) \le 0, i' \in K_l \quad (21\ )$$

T :

$$\mathrm{LBD} > \mathrm{UBD} \quad (22\ )$$

$$\mathrm{UBD} - \mathrm{LBD} < \varepsilon_1 \quad (22\ )$$

T :

$$\parallel \boldsymbol{\mu}^{k}_{a} / J^{k}_{a} \parallel_B \le \varepsilon_2 \quad (23\ )$$

$$\parallel \underline{\boldsymbol{u}}^{k}_{b} - \underline{\boldsymbol{u}}^{k}_{c} \parallel_B \le \varepsilon_3 \quad (23\ )$$

Algorithm 1: NSGBD with CVP for Problem (P1)

Step 1. D t_0,t_f , $t_0 < t_1 < t_2 < \ldots < t_N = t_f$, $\boldsymbol{u}(t)$ $\widehat{\boldsymbol{u}}_N$. F $\overline{\boldsymbol{u}}^{1}_{t}$ E . (7); $j = 1$, $j' = 1$, $K_{feas} = \emptyset$, $K_{infeas} = \emptyset$, $LBD = -\infty$; γ, ε_1, ε_2 ε_3.

Step 2. S P (P3) $\overline{\boldsymbol{u}}^{j}_{a} = \overline{\boldsymbol{u}}^{j}_{t}$. O - :

(1). P (P3) $\widehat{\boldsymbol{u}}^{j}_{N,a}$ J^{j}_{a}; - P (P4) $\underline{\boldsymbol{u}}^{j}_{a} = \overline{\boldsymbol{u}}^{j}_{a}$ $\overline{\boldsymbol{u}}^{j}_{a}$ $\widehat{\boldsymbol{u}}^{j}_{N,a}$ $\boldsymbol{\mu}^{j}_{a}$. UBD = J^{j}_{a}; O :

①. LBD > UBD. $\boldsymbol{\mu}^{l}_{a} = \gamma \boldsymbol{\mu}^{l}_{a}$, $l \in \Xi_a$.

②. UBD – LBD < ε_1. O :

(a). $|\Xi_b| + |\Xi_c| + |\Xi_d| = 0$ ($\overline{\boldsymbol{u}}^{j}_{a}$). I E . (23) - fi , $(\widehat{\boldsymbol{u}}^{j}_{N,a}, \overline{\boldsymbol{u}}^{j}_{a})$, $\boldsymbol{\mu}^{l}_{a} = \gamma \boldsymbol{\mu}^{l}_{a}$, $l \in \Xi_a$.

(b). $|\Xi_b| + |\Xi_c| + |\Xi_d| \ne 0$ ($\overline{\boldsymbol{u}}^{j}_{a}$). S P (P8) $\overline{\boldsymbol{u}}^{j}_{a}$ $\underline{\boldsymbol{u}}^{j}_{c}$. I E . (23) fi ($\underline{\boldsymbol{u}}^{j}_{b} = \overline{\boldsymbol{u}}^{j}_{a}$), $(\widehat{\boldsymbol{u}}^{j}_{N,a}, \overline{\boldsymbol{u}}^{j}_{a})$.

③. O .

$K_{feas} = \{K_{feas}, j\}$; $j = j + 1$.

(2). P P (P3) . S P (P5) $\overline{\boldsymbol{u}}^{j}_{t}$ $(\widehat{\boldsymbol{u}}^{j'}_{N,c}, \overline{\boldsymbol{u}}^{j'}_{c})$. S P (P6) $\overline{\boldsymbol{u}}^{j'}_{c}$ $(\widehat{\boldsymbol{u}}^{j'}_{N,c}, \overline{\boldsymbol{u}}^{j'}_{c})$ $\alpha = 0$ $\boldsymbol{\mu}^{j'}_{b}$. $K_{infeas} = K_{infeas}, j'$. $\overline{\boldsymbol{u}}^{j}_{t} = \overline{\boldsymbol{u}}^{j'}_{c}$, $j' = j' + 1$. R S 2.

Step 3. S P (P7) η^{j}_{b}, $\underline{\boldsymbol{u}}^{j}_{b}$, $\boldsymbol{v}^{j}_{a}$, $\boldsymbol{v}^{j}_{b}$, $\boldsymbol{v}^{j}_{c}$ $\boldsymbol{v}^{j}_{d}$; LBD = η^{j}_{b}, $\overline{\boldsymbol{u}}^{j}_{t} = \underline{\boldsymbol{u}}^{j}_{b}$. R S 2.

Remark 1. T fl NSGBD 13 F . 2. P (P3) (P4) fi $\overline{\boldsymbol{u}}^{j}_{a}$ - B . T P (P3) fi - P (P4) P (P3). P (P5) (P6) $\overline{\boldsymbol{u}}^{j}_{t}$ $\overline{\boldsymbol{u}}^{j}_{c}$ $\overline{\boldsymbol{u}}$ $\boldsymbol{\mu}^{j}_{b}$, $\overline{\boldsymbol{u}}$. P (P5) fi . P - (P6) P (P5) $\alpha = 0$. P (P7) LP . Ξ_a B . A , Ξ_a γ LBD . Ξ_b, Ξ_c Ξ_d

K_{infeas}, K_u K_l,
. E . (22)
LBD,
. E . (23) (23)
. P (P8)
E . (23),
. T NSGBD
S 2 A 1.

Remark 2. T NSGBD
: 1) μ_a^j ( )
13 $\overline{u}$; 2)
38 NSGBD, .

3.3. Novel implementation framework of optimal solution

A C P,

. M
. T

T

NCO - - ,

4. Hybrid parametric dynamic optimization of FCCU

4.1. Mathematical formulation of FCCU

A L . 7 , fl CO $u(t)$ $T_{ra\ sp}(t)$, $\bar{u}$ fl V CO M_{pro}. T CO 8 h = 480 2 h = 120 . F :

$$J\left(T_{ra\ sp}(t), \{V_i\}_{i=1}^{4}, M_{pro}\right) = \int_0^{480}\left(-\omega_{1d}F_d\left(t, T_{ra\ sp}(t), \{V_i\}_{i=1}^{4}, M_{pro}\right)\right. -\omega_{1n}F_n\left(t, T_{ra\ sp}(t), \{V_i\}_{i=1}^{4}, M_{pro}\right)\Big)dt + \sum_{i=1}^{4}\int_{120(i-1)}^{120i} f(V_i)dt + \omega_3 M_{pro} \quad (25)$$

ω_{1d}, ω_{1n} ω_3 , CO ; F_d F_n ; f $\{V_i\}_{i=1}^{4}$. T - - FCCU L . 7 . H , $T_{ra\ sp}(t)$ M_{pro} $\{V_i\}_{i=1}^{4}$ - E . (25), fi - . N . T - , .

T V , V ,

$$J\left(T_{ra\ sp}(t), V(t), M_{pro}\right) = \int_0^{480}\left(-\omega_{1d}F_d\left(t, T_{ra\ sp}(t), V(t), M_{pro}\right)\right. -\omega_{1n}F_n\left(t, T_{ra\ sp}(t), V(t), M_{pro}\right) + f\left(V(t)\right)\Big)dt + \omega_3 M_{pro} \quad (26)$$

C CO (M_{pro} = 4), . A T 1. T - L . 7 . T NSGBD PROMS/ OPT , MATLAB LP . M , 16 - . N , C 3, NSGBD C P.

4.2. Case 1: Combustion air as a continuous operation whereas CO promoter as a batch operation

I , M_{pro} - $T_{ra\ sp}$ V, L . 7 , C P - . H , 32 NLP . U NSGBD , , L T 2. U ε_1=1, ε_2= 0.1 ε_2= 0.1, NSGBD 5 . T $M_{pro}^{1,1} = 2$ $M_{pro}^{1,2} = 4$ fi , . T 5 $J^5 - LBD^5 \leq 1$ $\|\mu^5/J^5\| \leq$ 5×10^{-3} ( ), M_{pro}^{j} . C $M_{pro}^{5} = 3.1864$, J^5 = 1655.2¥. T - 5 F . 4.

A F . 4 4 , fl L CO , . 7 . A F . 4 4 , - O_2 fl - - . H , NCO- , - fl , . H , - - - . A L . 7 , - fl O_2 fl .

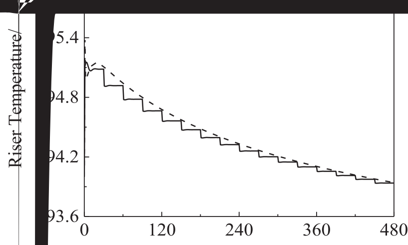
D fi $L = \{k \in N: \mu^k < 0\}$, $R = \{k \in N: \mu^k > 0\}$, $M_{pro}^{k} < M_{pro}^{*}$ $k \in L$ $M_{pro}^{k} > M_{pro}^{*}$ $k \in R$. H , -

$$M_{pro}^{*} \in \left(\max_{k \in L} M_{pro}^{k}, \min_{k \in R} M_{pro}^{k}\right) = (3.1864, 3.3440) \quad (27)$$

B S 3.3, NCO- , T 3.

A fi T 3, . M_{pro} = 3.1864 M_{pro} = 3.3440 , NCO- , . B T 3, M_{pro} 3.344, 3.45 . T M_{pro} = 3.4kg J = 2732.3¥

and4433284TD (001 Tf 2.767 0 T1TD ( )Tj /F1 1 Tf .7682 0 TD (R)Tj /F1 134111 Tf .7896 0 T]TJ1:)Tj /F1 Tf .7682 0 TD 73k426-2370-6.2(ð1.2nd44332844D (001-39m (p(aTJ905 1 Tf 3767 0 T18(for1Tj /F1 Tf .7682 0 TD



, -

. I , C P -
14 43 (21),
NSGBD 5 32 (16
). N NLP C P
NSGBD. A C P fi
fi - ,
NSGBD.

4.3. Case 2: Combustion air and CO promoter as batch operations

I , NLP -
16 , .
C , -
, L
T 4,
L
T 5,
T 6. U ε_1= 1, ε_2= 0.1 ε_3=
0.1, NSGBD 10 .
T 1, 2, 3 4. A
1, 4 2^5=32 . T
(2, 48, 48, 48, 48) (4, 48, 48, 48, 48) 1.1
1.2 B
. T (2, 50, 50, 50, 50) (4, 50, 50, 50, 50)
1.3 1.4 , -
,

B -
. A 2.1,
,
, B
. S -
, 2 (4, 50, 48, 48, 48) (4, 48, 50, 48, 48)
2.2 2.3. T ,
, B -
.
A 3.1 4.1,
3,
, . 3.2, 3.3 3.4 4.2, 4.3,
4.4, . B (3 4)
. F 5, -
. H , -
5 8, ,
. LBD > UBD, FCCU -
. F 5, B
1.2 4.2 (T 6), L -
γ = 1.16. T
8, 10. S
10 (T 6), (M_{pro}^{10},
$V_1^{10}, V_2^{10}, V_3^{10}, V_4^{10}$) . T P (P8)
E . (24) . H ,
NSGBD : $M_{pro}^{10} = 3.3223$, $V_1^{10} = 49.068$ $^3/$,
$V_2^{10} = 48.981$ $^3/$, $V_3^{10} = 48.931$ $^3/$, $V_4^{10} = 48.896$ $^3/$,
J^{10} = 1279.8¥, 10
F . 5.
A F . 5 ,
,
-
. H , NCO- -
, , - ,
. H , ,
- . M , fi
$P = \{k \in N : \mu_M^k > 0\}$, $\{N_i = \{k \in N : \mu_{Vi}^k < 0\}\}_{i=1}^4$. A μ_M $\{\mu_{Vi}\}_{i=1}^4$
,

Table 4
P .

I	4.4	5	6	7	8	9	10
M_{pro}( )	2.5865	2.4590	2.7855	2.8738	2.8890	3.2639	3.3223
μ_M	8.9744	−21.541	27.047	25.754	20.927	53.115	58.887
V_1(^{3}/)	49.034	49.061	49.064	49.064	49.065	49.068	49.068
μ_{V1}	−3480.3	−3515.4	−3471.9	−3479.8	−3490.8	−3480.0	−3468.3
V_2(^{3}/)	48.952	48.977	48.978	48.979	48.979	48.980	48.981
μ_{V2}	−3456.5	−3462.7	−3447.1	−3476.1	−3461.9	−3428.1	−3434.2
V							

:

$$M^*_{pro} \in (\quad _{k \in P} M^k_{pro}, M^{10}_{pro}) = (2.5865, 3.3223) \quad (28 \ )$$

$$V^*_1 \in (V^{10}_1, \quad _{k \in N_1} V^k_1) = (49.068, 49.072) \quad ^3/ \quad (28 \ )$$

$$V^*_2 \in (V^{10}_2, \quad _{k \in N_2} V^k_2) = (48.981, 48.983) \quad ^3/ \quad (28 \ )$$

$$V^*_3 \in (V^{10}_3, \quad _{k \in N_3} V^k_3) = (48.931, 48.932) \quad ^3/ \quad (28 \ )$$

$$V^*_4 \in (V^{10}_4, \quad _{k \in N_4} V^k_4) = (48.896, 48.897) \quad ^3/ \quad (28 \ )$$

T NCO- T 7. A fi T 7, . $(M^{10}_{pro}, V^{10}_1, V^{10}_2, V^{10}_3, V^{10}_4)$, NSGBD NCO- . B T 7, : $M_{pro} \in (3.3223, 3.3223)$, $V_1 \in (49.076, 49.080)$ $^3/$, $V_2 \in (48.988, 48.992)$ $^3/$, $V_3 \in (48.938, 48.942)$ $^3/$, $V_4 \in (48.904, 48.908)$ $^3/$. T $M_{pro} = 3.3223$, $V_1 = 49.078$ $^3/$, $V_2 = 48.990$ $^3/$, $V_3 = 48.940$ $^3/$, $V_4 = 48.906$ $^3/$ $J = 1890.4$¥, NCO- F . 5. A

F . 5 , NCO- . T fl 2 8 , fi 0.5 (F . 5). M , .

F C 3 4, NCO- , - T 8. C C 2 1 4 3, - .

T

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